Solving Simultaneous Equations

(1) **Conceptual Atomisation**

- Complete atomisation of all related concepts, both explicit and implicit

(2a) **Routine Atomisation**

- Example atomisation for one cognitive routine, including fully overtised prompts and implicit concepts

(2b) **Full Chain**

- Full chain of atoms required to move from an initial prompt to final response in a mostly covertised manner

(2c) **Sub Chains**

- Examples of sub chains that can be practised before attempting the full chain

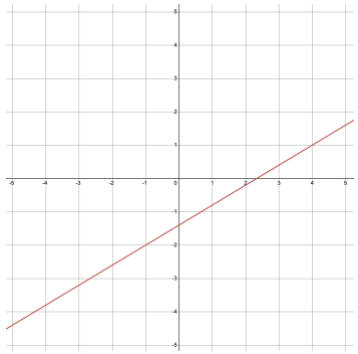
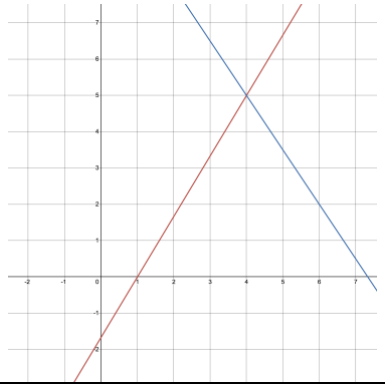
(3) **Atomic Instruction**

- Example atomic instruction, including initial instruction, initial testing, and expansion sequence, for individual concept atoms

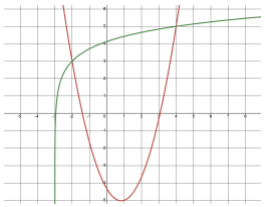
Conceptual Atomisation

#	Example Prompt	Example Response	Type	Notes
1	Solve $19x = 76$	$x = 4$	Routine	**Pre-requisite** Solve one-step equations
2	Substitute: $x = 4, y = 5$ $5x - 3y$	$5(4) - 3(5)$	Transformation	**Pre-requisite** Substitute into two or more unknowns when explicitly told what x and y are equal to
3	Substitute: $(4, 5)$ $5x - 3y$	$5(4) - 3(5)$	Transformation	**Pre-requisite** Substitute into two or more unknowns when given x and y in coordinate form
4	Show that $(4, 5)$ is a solution to: $5x - 3y = 5$	$5(4) - 3(5) = 5$ $20 - 15 = 5$ $5 = 5$	Routine	**Pre-requisite** Show that (x, y) is a solution to an equation
5	Can this equation be solved? $5x - 3y = 5$ How do you know?	No Because it has more than one unknown	Categorical	Identify when equations are unsolvable

6	Add the equations: $5x + 4y = 20$ $2x + 6y = 5$	$7x + 10y = 25$	Transformation	Add or Subtract two or more equations
7	Does this equation have an infinity of solutions? $5x - 3y = 5$	Yes	Categorical	Identify when equations have an infinity of solutions
8	Find four solutions to this equation: $x + y = 10$	Example responses: $(-2, 12)$ $\left(\frac{1}{2}, \frac{19}{2}\right)$ $(0, 10)$ $(3, 7)$	Routine	Find some solutions to an equation that has an infinity of solutions
9	Add or subtract? $9x + 6y = 66$ $10x - 6y = 10$	Add	Categorical	Decide whether to add or subtract a pair of equations to eliminate an unknown

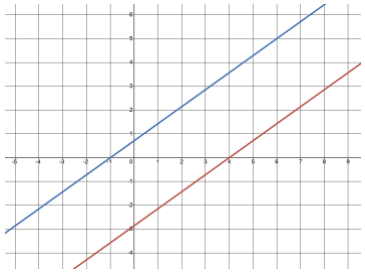
10	Does the equation represented by this graph have an infinity of solutions? 	Yes	Categorical	Identify when equations have an infinity of solutions from their graphs
11	What is the solution to both equations? $5x - 3y = 5$ $3x + 2y = 22$ 	$(4, 5)$ $x = 4$ and $y = 5$	Transformation	Identify the (x, y) solution to a pair of linear equations based on their graphs

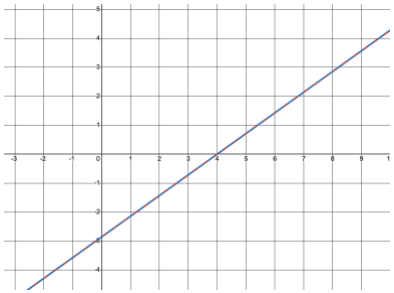
12	"What is the solution to these equations?" $5x - 3y = 5$ $3x + 2y = 22$ $2x + y = 13$ $x - 5y = -21$ 	$(4, 5)$ $x = 4$ and $y = 5$	Transformation	Identify the (x, y) solution to three or more linear equations based on their graphs
13	"If simultaneous equations have more than one solution, what do we call them?" "A 'set of solutions' means...?"	"A solution set." "That there is more than one solution to the equations."	Fact	Know that when there is a single, unique, solution to one or more equations, we refer to it as '**the solution**,' even if the solution includes more than one variable. When there is more than one solution, more than one possible value for at least one of the variables, we refer to it as '**the solution set**,' or '**set of solutions**.'

14	"Do these equations have a unique solution, or a set of solutions?" $5x - 3y = 5$ $3x + 2y = 22$ Solution(s): $x = 4$ and $y = 5$	Unique solution	Categorical	Distinguish between **solution sets** and **unique solutions**
15	"What are the solutions to these equations?" $y = x^2 - \frac{5}{3}x - \frac{13}{3}$ $y = \frac{2}{\ln(7)} \ln(x+3) + 3$ 	$(4, 5)$ And $(-2, 3)$	Transformation	Identify the (x, y) solutions to two or more linear and non-linear equations based on their graphs
16	"Do these equations have a common coefficient?" $5x - 3y = 5$ $3x + 2y = 22$	No	Categorical	Identify pairs of equations with a common coefficient

17	"Multiply to create a common coefficient." $5x - 3y = 5$ $3x + 2y = 22$	$3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	Transformation	Cross multiply two equations by the leading coefficient to get a common coefficient
18	"Multiply to create a common coefficient." $5x - 3y = 5$ $3x + 2y = 22$	$2(5x - 3y) = 2(5)$ $3(3x + 2y) = 3(22)$	Routine	Multiply two equations to get a common coefficient in the most efficient way possible, resulting in lowest possible values
19	"Expand the brackets." $3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	$15x - 9y = 15$ $15x + 10y = 110$	Transformation	**Pre-requisite** Expand single brackets
20	"Solve these equations together." $5x - 3y = 5$ $3x + 2y = 22$	[working] $x = 4$ and $y = 5$	Routine	Respond correctly to the instruction to 'solve the equations **together**'

21	“What technical word do we use to mean ‘together’?” “Simultaneous means…?”	“Simultaneous” “Together”	Fact	Know that ‘simultaneous’ means ‘together’
22	“Are these equations simultaneous?” $5x - 3y = 5$ $3a + 2b = 22$	No	Categorical	Identify when two or more equations are simultaneous, as opposed to non-simultaneous
23	“Are these equations fully, partially, or non-simultaneous?” $5x - 3y = 5$ $3a + 2x = 22$	Partially	Categorical	Identify when two or more equations are partially simultaneous, as opposed to non-simultaneous, or fully simultaneous
24	“Solve the simultaneous equations.” $5x - 3y = 5$ $3x + 2y = 22$	[working] $x = 4$ and $y = 5$	Routine	Respond correctly to the instruction to ‘solve the **simultaneous** equations.’

25	"Solve:" $5x - 3y = 5$ $3x + 2y = 22$	[working] $x = 4$ and $y = 5$	Routine	Solve a pair of linear simultaneous equations by elimination without prompting.
26	"Do these simultaneous equations have a solution?" $5x - 7y = 20$ $10x - 14y = -10$ 	No	Categorical	Identify when two linear simultaneous equations have no solutions, based on their graphs

27	"Do these simultaneous equations have an infinity of solutions?" $5x - 7y = 20$ $10x - 14y = 40$ 	Yes	Categorical	Identify when two linear simultaneous equations have an infinity of solutions, based on their graphs
28	"Show that these simultaneous equations have no solutions." $4x + 6y = 15$ $2x + 3y = 6$	$4x + 6y = 15$ $4x + 6y = 12$ $0 \neq 3$ or $15 \neq 12$	Routine	Show algebraically that two linear simultaneous equations have no solutions

29	"Show that these simultaneous equations have an infinity of solutions." $4x + 6y = 12$ $2x + 3y = 6$	$4x + 6y = 12$ $4x + 6y = 12$ $0 = 0$ or $12 = 12$	Routine	Show algebraically that two linear simultaneous equations have an infinity of solutions
30	"Substitute for x in the expression:" $x = 3 + 10y$ $5x - 3y$	$5(3 + 10y) - 3y$	Transformation	**Pre-requisite** Substitute one expression into another
31	"Rearrange to make x the subject." $5x - 50y = 15$	$5x = 15 + 50y$ $x = 3 + 10y$	Routine	**Pre-requisite** Rearrange equations to make one variable the subject of another
32	"Solve by substitution." $y = 3x + 5$ $2x - y = 1$	[working] $(-6, -13)$ $x = -6$ and $y = -13$	Routine	Solve a pair of simultaneous linear equations by substitution, presenting the solution as either a coordinate pair or as explicit variable assignment

33	"Would these be better solved by elimination or substitution?" $y = 3x + 5$ $2x - y = 1$	Substitution	Categorical	Identify whether elimination or substitution would be a more efficient method of solving
34	Solve: $y = x^2 + 2x - 3$ $y = 3x + 1$	$\left(\frac{1+\sqrt{17}}{2}, \frac{5+3\sqrt{17}}{2}\right)$ or $\left(\frac{1-\sqrt{17}}{2}, \frac{5-3\sqrt{17}}{2}\right)$	Routine	Solve a pair of simultaneous equations where one is quadratic
35	Solve: $y = x^2 + 2x$ $y = x^2 + x + 2$	$(2, 8)$ $x = 2$ and $y = 8$	Routine	Solve a pair of simultaneous equations where both are quadratic

Routine Atomisation – 25 – Solve Simultaneous Equations by Elimination

#	Example Prompt	Example Response	Type	Notes
1	"Do these equations have a common coefficient?" $5x - 3y = 5$ $3x + 2y = 22$	No	Categorical	Identify pairs of equations with a common coefficient
2	"Multiply to create a common coefficient." $5x - 3y = 5$ $3x + 2y = 22$	$3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	Transformation	Cross multiply two equations by the leading coefficient to get a common coefficient
3	"Expand the brackets." $3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	$15x - 9y = 15$ $15x + 10y = 110$	Transformation	**Pre-requisite** Expand single brackets
4	Add or subtract? $15x - 9y = 15$ $15x + 10y = 110$	Subtract	Categorical	Decide whether to add or subtract a pair of equations to eliminate an unknown

5	Subtract the equations: $15x - 9y = 15$ $15x + 10y = 110$	$-19y = -95$	Transformation	Add or Subtract two or more equations
6	Solve $-19y = -95$	$y = \frac{-19}{-95}$ $y = 5$	Routine	**Pre-requisite** Solve one-step equations
7	Substitute: $y = 5$ $5x - 3y = 5$	$5x - 3(5) = 5$	Transformation	**Pre-requisite** Substitute into two or more unknowns when explicitly told what x and y are equal to
8	Solve $5x - 3(5) = 5$	$5x - 15 = 5$ $5x = 20$ $x = 4$	Routine	**Pre-requisite** Solve linear equations
9	Show that $(4, 5)$ is a solution to: $3x + 2y = 22$	$3(4) + 2(5) = 22$ $12 + 10 = 22$ $22 = 22$	Routine	**Pre-requisite** Show that (x, y) is a solution to an equation

Full Chain – 25 - Solve Simultaneous Equations by Elimination

#	Example Prompt	Type
0	Solve the simultaneous equations: $5x - 3y = 5$ $3x + 2y = 22$	Initial Prompt
1	$3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	Transformation
2	$15x - 9y = 15$ $15x + 10y = 110$	Transformation
3	$15x - 9y = 15$ $15x + 10y = 110 \quad -$ _____	Categorical
4	$-19y = -95$	Transformation
5	$y = \frac{-19}{-95}$ $y = 5$	Routine
6	$5x - 3y = 5$ $5x - 3(5) = 5$	Transformation

7	$5x - 15 = 5$ $5x = 20$ $x = 4$	Routine
8	**CHECK:** $3x + 2y = 22$ $3(4) + 2(5) = 22$ $12 + 10 = 22$ $22 = 22$	Routine

Sub Chains – 25 – Solve Simultaneous Equations by Elimination

Since there are 8 atoms in the full chain, there are 35 possible sub chains.

Three examples are given below

Example 1

#	Example Prompt	Type
0	What sum would eliminate a variable from these simultaneous equations? $5x - 3y = 5$ $3x + 2y = 22$	Initial Prompt
1	$3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	Transformation
2	$15x - 9y = 15$ $15x + 10y = 110$	Transformation
3	$15x - 9y = 15$ $15x + 10y = 110 \quad -$ $\_\_\_\_\_$	Categorical

Example 2

5	Here are two simultaneous equations: $5x - 3y = 5$ $3x + 2y = 22$ Given that: $y = 5$ What is x? Check that your solution is correct.	Initial Prompt
6	$5x - 3y = 5$ $5x - 3(5) = 5$	Transformation
7	$5x - 15 = 5$ $5x = 20$ $x = 4$	Routine
8	**CHECK:** $3x + 2y = 22$ $3(4) + 2(5) = 22$ $12 + 10 = 22$ $22 = 22$	Routine

Example 3

1	What is the value of y? $3(5x - 3y) = 3(5)$ $5(3x + 2y) = 5(22)$	Initial Prompt
2	$15x - 9y = 15$ $15x + 10y = 110$	Transformation
3	$15x - 9y = 15$ $15x + 10y = 110 \quad -$ _____	Categorical
4	$-19y = -95$	Transformation
5	$y = \frac{-19}{-95}$ $y = 5$	Routine

Atomic Instruction – 6 – Add or Subtract Equations

#	Example Prompt	Example Response	Type	Notes
6	Add the equations: $5x + 4y = 20$ $2x + 6y = 5$	$7x + 10y = 25$	Transformation	Add or Subtract two or more equations

Introduction

"When you first went to primary school, we taught you how to add numbers."

$$\begin{array}{r} 5 \\ 2+ \\ \hline 7 \end{array}$$

"When you came to use in secondary school, we then taught you how to add together letters, variables, unknowns."

$$\begin{array}{r} 5x \\ 2x+ \\ \hline 7x \end{array}$$

"I'm now going to show you how to add together *entire equations*!"

Initial Instruction

"My turn first."

"I add these equations together, like this..."

$$\begin{array}{r} 5x + 4y = 20 \\ 2x + 6y = 5 \quad + \\ \hline 7x + 10y = 25 \end{array}$$

"...because five x plus two x is seven x, four y plus six y is ten y, and twenty plus five is twenty-five."

Initial Testing

"Your turn. I'm going to change just one thing... watch what I change... Now, add these equations..."

Task Sequence

#	Task	Correct Response
1	$5x + 10y = 20$ $2x + 6y = 5$	$7x + 16y = 25$
2	$3x + 10y = 20$ $2x + 6y = 5$	$5x + 16y = 25$
3	$3x + 10y = 20$ $2x + 3y = 3$	$5x + 13y = 23$

Expansion Sequence

Task Sequence

#	Task	Correct Response	New Idea
1	$3a + 10b = 20$ $2a + 3b = 3$	$5a + 13b = 23$	It doesn't have to be x and y
2	$3a + 10b = 20$ $2a + 3b = 3$ $10a + 5b = 7$	$15a + 18b = 30$	It's possible to add more than two equations
3	$3a + 10b = 20$ $2a - 3b = 3$	$5a + 7b = 23$	We can add negative coefficients in equations
4	$3a + 10b = 20$ $2a - 10b = 3$	$5a + 0b = 23$ or $5a = 23$	Sometimes, one variable might be eliminated

5	$10x + 5y = 12$ $2y + 3x = 8$	$13x + 7y = 20$	Add the corresponding coefficients, not the coefficients that are vertically aligned
6	$10x + 5y = 12$ $2y = 8$	$10x + 7y = 20$	Sometimes one variable might not appear in the other equation
7	$10x = 12$ $2y = 8$	$10x + 2y = 20$	Sometimes the equations might appear independent (**Note:** technically this addition is only valid if it is stated that the two equations are part of the same system. In that case, this addition is not **useful**, but it is **valid**.)
8	$5 + 10 = 15$ $2 + 8 = 10$	$7 + 18 = 25$	It's possible to add equations with no variables or unknowns. Equality of the statements will be maintained.
9	$10x + 5y = 12$ $2a + 3b = 8$	$10x + 5y + 2a + 3b = 20$	Sometimes the equations might appear independent, with more than one variable each. (**Note:** technically this addition is only valid if it is stated that the two equations are part of the same system. In that case, this addition is not **useful**, but it is **valid**.)

10	$10x + 5y = 12$ $2a + 3b = 8 + 3x$	$10x + 5y + 2a + 3b = 20 + 3x$	Addition of partially simultaneous equations, where the shared variables are on different sides of the equals sign
11	$-7x + 2y = 12$ and $2y + 3x = -8$	$-4x + 7y = 4$	The equations might not be presented in vertical alignment
12	$5x + 3y = 15$ $+ \quad + \quad =$ $12x \quad 8y \quad 3$	$7x$ $11y \ldots 18$	Using the information given, and knowledge of equation addition, to work out unknown information
13	$10p + 5q = 12$ $2p + 1q = 8$	$12p + 6q = 20$	Addition where a variable has an explicit coefficient of 1
14	$10p + 5q = 12$ $2p + q = 8$	$12p + 6q = 20$	Addition where a variable has an implicit coefficient of 1 (secret one)